Dr. Marques Sophie
Office 519

Algebra II

Spring Semester 2014

marques@cims.nyu.edu

Problem Set # 8

Exercise 1:
Section 4.2 [F] 3., 9., 10. .
Exercise 2:
We say that a matrix $A \in M_n(K)$ is singular, if $Ker(L_A) \neq \{0\}$.

1. Prove that if A, $B \in M_n(K)$ and either matrix is singular. Then AB is singular.
2. Prove that if A and B are both non-singular, so is AB.

Exercise 3:
Let $T : V \to W$ be a linear operator between finite dimensional vector spaces and let $\mathcal{X} = \{e_1, ..., e_n\}$, $\mathcal{N} = \{f_1, ..., f_m\}$ be bases in V, W.
We have defined $rank(T) = dim(R(T))$. If $A = [T]_{\mathcal{N},\mathcal{X}}$. Prove the identity
$rank(T)$ =rank of the linear operator $L_A : K^n \to K^m$.

Exercise 4:
Prove that the following statement are equivalent:

1. $det(A) \neq 0$;
2. A has multiplicative inverse A^{-1} such that $A^{-1}A = AA^{-1} = Id$.
3. $L_A : K^n \to K^n$ is an invertible linear mapping (one-to-one and onto).

Hint: You can prove $(a) \Rightarrow (b) \Rightarrow (c) \Rightarrow (a)$.

Exercise 5:
If A is an $n \times n$ matrix, the following conditions are equivalent:

1. $det(A) \neq 0$ (i.e. A is a nonsingular matrix and L_A is invertible);
2. the rows R_1,...,R_n are linearly independent in K^n;
3. the columns C_1,..., C_n are linearly independent in K^n.